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THE STRESSED STATE OF A BOX-LIKE SHELL REINFORCED BY A PAIR OF SYMMETRIC INCLUSIONS PARALLEL TO THE EDGE OF THE SHELL†

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The problem of the stressed state of an infinite box-like shell of rectangular profile is solved. The shell is reinforced by two absolutely rigid thin inclusions placed on opposite sides and parallel to the shell edges. This problem can be reduced [1] to that concerned with the joint plane and bent state of a plate with defects, the role of the latter being played by the shell edges and the inclusions. On applying a semi-infinite cosine Fourier transform the problem can be reduced to a system of two integral equations with respect to the jumps of the generalized transverse force and shear stresses, which has no solutions in terms of integrable functions [2–4]. The solution is sought in the space of functions having non-integrable singularities by applying regularization methods for divergent integrals [4]. Diagrams of the dependence of inclusion settling on the length of inclusions and the geometric dimensions of the cross-sections of the shell are constructed.

We consider the problem of the stressed state of a box-like shell of infinite length and rectangular cross-section reinforced by two symmetrically placed parallel inclusions. Concentrated forces are applied to the centres of the inclusions (see Fig. 1).

The problem can be reduced [1] to solving the system of equations

$$\Delta^2 w(x,y)=0,\quad \Delta^2\sigma_x(x,y)=0,\quad -a<x<b,\quad x\neq 0,\quad |y|<\infty \tag{1}$$

satisfying the conditions

$$\begin{aligned}&\langle \upsilon\rangle=\langle\tau_{xy}\rangle=\langle\varphi_x\rangle=\langle M_x\rangle=0\\&\langle u\rangle=-(w_+ + w_-),\quad \langle w\rangle=u_+ + u_-\\&\langle\sigma_x\rangle=-h^{-1}[(V_x)_+ + (V_x)_-],\quad \langle V_x\rangle=h[(\sigma_x)_+ + (\sigma_x)_-]\end{aligned} \tag{2}$$

on the shell edges, the symmetry conditions

$$V_y=\varphi_y=\upsilon=\tau_{xy}=0,\quad y=0,\quad -a<x<b \tag{3}$$

$$V_x=\varphi_y=u=\tau_{xy}=0,\quad x=-a(|y|>c),\quad x=b \tag{4}$$

the conditions

$$u(-a,y)=\varphi_x(-a,y)=0,\quad \upsilon(-a,y)=0,\quad w(-a,y)=\delta,\quad |y|<c \tag{5}$$

on the inclusions, and the equilibrium conditions

$$\int_{-c}^{c} V_x(-a,y)dy=-\frac{P}{2} \tag{6}$$

for the inclusions. The remaining equilibrium conditions are satisfied automatically by symmetry, the displacement δ being unknown in advance. Here we use the notation of [1].

We introduce the unknown functions

$$\chi(y)=V_x(-a,y),\quad \mu(y)=\tau_{xy}(-a,y) \tag{7}$$

which are non-zero only on the inclusion $|y|<c$.

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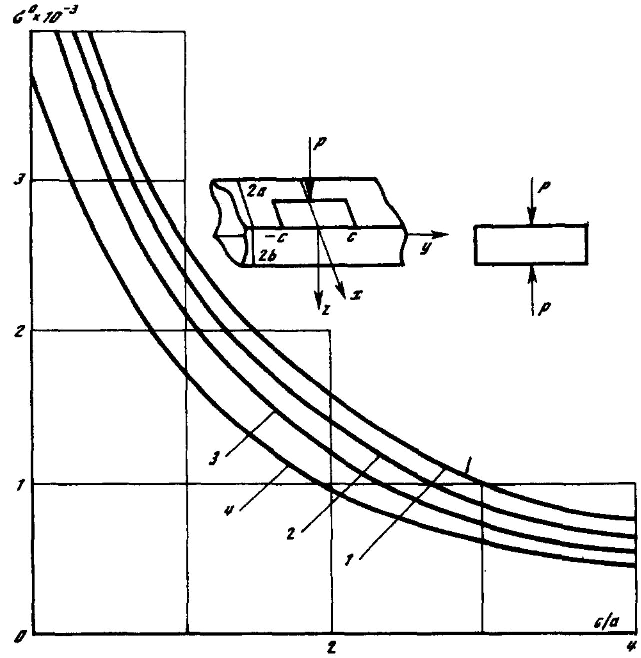

Fig. 1.

Applying the extended method of integral transforms [5], we arrive at a one-dimensional discontinuous boundary-value problem for Fourier transforms, the solution of which can be written in the form [1, 5]

$$\begin{aligned} &f_\alpha^\pm(x) = -\mu_\alpha k_\mu^\pm(\alpha,x) - X_\alpha k_\chi(\alpha,x) \\ &k_\mu^+(\alpha,x) = \alpha R_2^+ G_\alpha(x,-a) - f_{3\mu}^+ T_0^+ G_\alpha + f_{0\mu}^+ T_3^+ G_\alpha \end{aligned} \tag{8}$$

$$\begin{aligned} &k_\mu^-(\alpha,x) = -f_{3\mu}^- T_0^- G_\alpha + f_{0\mu}^- T_3^- G_\alpha \\ &k_\chi^+(\alpha,x) = -f_{3\chi}^+ T_0^+ G_\alpha + f_{0\chi}^+ T_3^+ G_\alpha \\ &k_\chi^-(\alpha,x) = D^{-1} G_\alpha(x,-a) - f_{3\chi}^- T_0^- G_\alpha + f_{0\chi}^- T_3^- G_\alpha \end{aligned} \tag{9}$$

Here $f_{j\chi}^\pm$ $(j = 0, 3)$ is the solution of the system of linear algebraic equations

$$\begin{Vmatrix} 1 & 0 & -C_{33}^- & C_{30}^- \\ 0 & \alpha^{-4}D & -C_{03}^- & C_{00}^- \\ -C_{33}^+ & C_{30}^+ & -\alpha^4 D & 0 \\ -C_{03}^+ & C_{00}^+ & 0 & 1 \end{Vmatrix} \begin{Vmatrix} f_{0\chi}^+ \\ f_{3\chi}^+ \\ f_{0\chi}^- \\ f_{3\chi}^- \end{Vmatrix} = \begin{Vmatrix} D^{-1} H_3^- G_\alpha(x,-a) \\ D^- H_0^- G_\alpha(x,-a) \\ 0 \\ 0 \end{Vmatrix} \tag{10}$$

$$C_{ij}^\pm = H_i^\pm T_j^\pm [G_\alpha^\pm]$$

and $f_{j\mu}^\pm$ $(j = 0, 3)$ is the solution of (10) with right-hand side

$$\left\| 0,\ 0,\ \alpha H_3^+ R_2^+ G_\alpha(x,-a),\ -\alpha H_0^+ R_2^+ G_\alpha(x,-a) \right\|^{\mathrm{T}} \tag{11}$$

where $G_\alpha(x, \xi)$ is Green's function of the boundary-value problem

$$L^2 u(x) = 0, \quad u' = u''' = 0, \quad x = -a, b \tag{12}$$

To find the unknown functions $\chi(y)$ and $\mu(y)$ we use the conditions of [5], the first two of which are satisfied automatically by the choice of $G_\alpha(x, \xi)$, and the last two of which lead to a system of two integral equations for the desired functions. We separate the singular parts in the kernels of these two equations using the integrals

$$\int_0^\infty \cos\alpha y \sin\alpha y \frac{d\alpha}{\alpha} = \frac{1}{2}\ln\left|\frac{y+\eta}{y-\eta}\right|$$

$$\int_0^\infty \left[\cos\alpha t - \theta(A-\alpha) + \frac{1}{2}\alpha^2 t^2 e^{-\alpha b}\right]\frac{d\alpha}{\alpha} = \frac{1}{2}t^2\ln|t| - t^2\left(\frac{3}{4}+\frac{1}{2}\ln b\right) + \frac{1}{2A^3} \tag{13}$$

$$A = \text{const} > 0$$

the latter being obtained by integrating the improper integral

$$\int_0^\infty \frac{e^{-\alpha b} - \cos\alpha t}{\alpha} d\alpha = \ln|t| - \ln b, \quad b > 0$$

twice with respect to t. As a result, we obtain a system of integral equations of the form

$$\int_{-1}^{1} \left\| \begin{matrix} (y-\eta)^2 \ln|y-\eta| & 0 \\ 0 & \ln\frac{1}{|y-\eta|} \end{matrix} \right\| \left\| \begin{matrix} \chi(\eta) \\ \mu(\eta) \end{matrix} \right\| d\eta +$$

$$+\int_{-1}^{1} \left\| \begin{matrix} K_{11}(y',\eta) & K_{12}(y,\eta) \\ K_{21}(y,\eta) & K_{22}(y,\eta) \end{matrix} \right\| \left\| \begin{matrix} \chi(\eta) \\ \mu(\eta) \end{matrix} \right\| d\eta \left\| \begin{matrix} 4\pi D\delta \\ 0 \end{matrix} \right\| \tag{14}$$

$$K_{11}(y,\eta) = A^{-2} - (y-\eta)^2(3/2 + \ln b) +$$

$$+\int_0^\infty [k_{11}(\alpha)\cos\alpha y \cos\alpha\eta + 2\alpha^{-3}\theta(A-\alpha) + \alpha^{-1}(y-\eta)^2 e^{-\alpha b}]d\alpha$$

$$K_{12}(y,\eta) = \int_0^\infty k_{12}(\alpha)\cos\alpha y \sin\alpha\eta d\alpha, \quad K_{21}(y,\eta) = \int_0^\infty k_{21}(\alpha)\sin\alpha y \cos\alpha\eta d\alpha$$

$$k_{11}(\alpha) = 4\left\{\left[-\frac{a+b}{2\alpha}LG_\alpha(-a,b) + G_\alpha(-a,b)\right]e^{-\alpha(a+b)} + D[f_{0\chi}^- w_{\alpha 3} - f_{3\chi}^- w_{\alpha 0}]\right.$$

$$k_{12}(\alpha) = 4D[f_{0\mu}^- w_{23} - f_{3\mu}^- w_{\alpha 0}], \quad k_{21}(\alpha) = -2\kappa^{-1}E[f_{0\chi}^+ \upsilon_{\alpha 3} - f_{3\chi}\upsilon_{\alpha 0}]$$

$$k_{22}(\alpha) = -(3-\nu)^{-1}\{[4-(1+\nu)\alpha(a+b)]LG_\alpha(-a,b) + 2(1+\nu)\alpha^2 G_\alpha(-a,b)\} -$$

$$-2\kappa^{-1}E[f_{0\mu}^+ \upsilon_{\alpha 3} - f_{3\mu}^+ \upsilon_{\alpha 0}],$$

$$w_{\alpha j} = T_j^-[G_\alpha(-a,t)], \quad \upsilon_{\alpha j} = (-\alpha^3 E)^{-1} R_2^+ T_j^+ G_\alpha(-a,t)], \quad j = 0,3$$

As was shown in [2-4], the first of integral equations (14) has no solutions in terms of integrable functions. There it was proposed to seek the solution in the space of functions having non-integrable singularities of the form $(1-y^2)^{-3/2}$ and to use regularization methods for divergent integrals.

We will use the method of orthogonal polynomials [5] to solve (14). We seek $\mu(y)$ as a series in terms of Chebyshev polynomials of the first kind and $\chi(y)$ in terms of polynomials of a special form $\pi_n(y)$ [4]

$$\mu(\eta) = \sum_{l=0}^\infty \mu_l \frac{T_{2l+1}(\eta)}{(1-\eta^2)^{1/2}}, \quad \chi(\eta) = \sum_{l=0}^\infty \chi_l \pi_{2l}(\eta) \tag{15}$$

$$\pi_0(t) = (1-t^2)^{-1/2}, \quad \pi_{2n}(t) = \frac{2\sqrt{\pi}(2n)! P_{2n}^{-3/2,-3/2}(t)}{\Gamma(2n-1/2)(1-t^2)^{3/2}}$$

As a result, we obtain an infinite system of linear algebraic equations for the coefficients of the expansion (15).

Substituting (15) into the equilibrium conditions (6) and taking (7) into account, we find that

$$\delta = \frac{P}{2\Psi_{00}\chi_0} \tag{16}$$

where χ_j is a solution of the algebraic system obtained for $\delta = 1$.

In Fig. 1 we show graphs of the inclusion settling $\delta^0 = \delta EhP^{-1}$ as a function of the inclusion length c/a for $\nu = 0.3$, $h/a = 0.01$ and various a/b. For $c \to 0$ the inclusion settling approaches the deflection obtained under the point where the force is applied in the problem of a shell loaded by two symmetric concentrated forces.

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